Engineering Notes

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Guidance of a Bank-to-Turn Missile with Altitude Control Requirements

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Nomenclature

A	$= \lvert i_v \times i_r \rvert = \lvert \sin \beta \rvert$
$\mathbf{a}\cdot\mathbf{b}, \mathbf{a}\times\mathbf{b}$	= dot product and cross product of vectors **a** and **b**
$\mathbf{a}_c$	= commanded acceleration vector
$\mathbf{a}_g$	= acceleration vector required by guidance law
$\mathbf{a}_h$	= missile roll plane component of acceleration vector required by altitude control system $= a_h v$
a_l, a_m, a_n	= components of $\mathbf{a}_g$ along body axes
h	= missile altitude
(i, j, k), (l, m, n), (i_v, j_v, k_v)	= unit vectors defining inertial axes, missile-fixed body axes (roll, pitch, yaw) and missile velocity axes, respectively
P, Q, R	= components of missile angular velocity vector along body axes
$\mathbf{r}$	= line of sight vector from missile to target $= ri_r$
$\mathbf{V}$	= missile velocity vector $= Vi_v$
β	= look angle between **V** and **r**, heading error angle
$\dot\beta$	= rate of change of heading error angle
η_1, η_2, η_3	= components of $\mathbf{a}_g$ along velocity axes
μ	= unit vector defining intersection of missile roll plane and local vertical plane $= (i \times l)/\lvert i \times l \rvert$
v	= unit normal vector to plane defined by **V** and $\mathbf{r} = i_v \times i_r / A$
ϕ, ϕ_c	= missile roll angle, commanded roll angle
$\mathbf{\Omega}_c$	= angular velocity vector required by guidance law $= \Omega_c v$
$\lvert\mathbf{a}\times\mathbf{b}\rvert$	= absolute value or magnitude of $\mathbf{c} = \mathbf{a}\times\mathbf{b}$

Introduction

A MISSILE with a preferred maneuver plane must "bank-to-turn" in order to respond to guidance commands when homing on a target. This Note reports on certain unclassified aspects of a study of such a missile trying to hit a slower-moving surface target with missile-to-target speed ratio exceeding six. Furthermore, the missile must be able to maintain or control to a certain altitude profile while carrying out the maneuver required by the guidance system.

The missile guidance and control problem is, essentially, three-dimensional. Because of the favorable missile-to-target speed ratio and maneuver capabilities in this application, a simple heading error guidance scheme was used rather than three-dimensional proportional navigation.[1] The acceleration commands thus generated were combined with the altitude control signal in a novel manner to give resultant commands to the missile roll and pitch autopilots. The scheme was found to be effective.

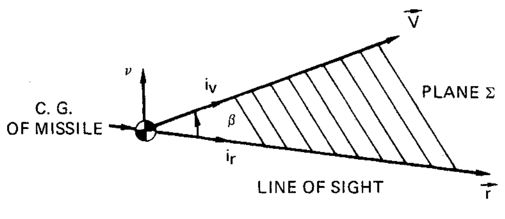

Fig. 1 Definition of heading error angle β.

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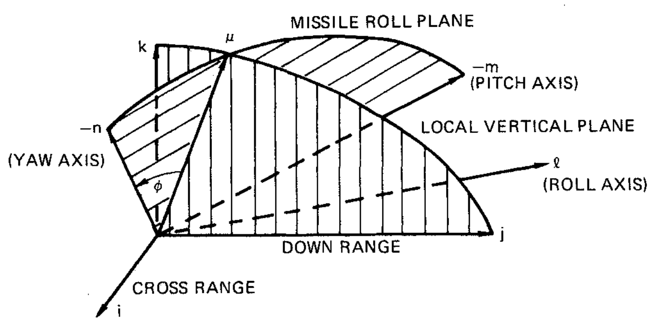

Fig. 2 Definition of roll angle ϕ.

Guidance Law

The missile velocity vector **V** and the line of sight vector **r**, as shown in Fig. 1, define a unique plane Σ, unless they are collinear. For zero heading error β, **V** should point along the line of sight. Rotating **V** about the normal to Σ gives the required angular velocity command $\mathbf{\Omega}_c$. The magnitude Ω_c may be made proportional to β or, for faster moving targets, to a suitably weighted sum of β and $\dot\beta$. The direction of $\mathbf{\Omega}_c$ is given by v. Thus, $\eta_1 = 0$ and the components of the commanded acceleration $\mathbf{a}_g$ along the j_v and k_v axes are, respectively

$$\eta_2 = V\Omega_c(j_v \cdot i_r)/A \tag{1}$$

$$\eta_2 = V\Omega_c(k_v \cdot i_r)/A \tag{2}$$

These can be transformed into acceleration commands along the missile body axes.[2] The command a_l along the roll axis is generally small for small angles of attack and may be neglected. The other two commands a_m and a_n become the outputs of the guidance computer. Thus, the resultant guidance command vector will lie in the missile roll plane.

True Roll Angle

Referring to Fig. 2, the true roll angle of the missile is defined as the angle ϕ between the $-n$ axis (yaw axis) and the vector μ. It follows that

$$\dot\phi = P - [c_{11}/(1-c_{11}^2)][c_{21}Q + c_{31}R] \tag{3}$$

from which ϕ can be obtained by integration from a given initial value. The direction cosines c_{ij} are obtained in the usual way from the angular rates P, Q, R.

Bank-to-Turn Maneuver

It is assumed that the missile has little yaw maneuver capability so the preferred maneuver plane is the pitch plane. Thus, in normal flight with no guidance command, the missile would maintain or change altitude with the pitch plane coinciding with the local vertical plane. On receipt of a guidance command, however, the missile must roll until the required acceleration vector lies in the pitch plane, as shown in Fig. 3. In this figure,

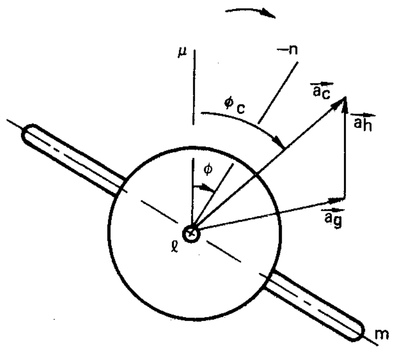

Fig. 3 Bank-to-turn commands.

the missile roll plane is the plane of the page. The required acceleration command vector is

$$\mathbf{a}_c = \mathbf{a}_h + \mathbf{a}_g \tag{4}$$

where

$$\mathbf{a}_g = \mathbf{a}_m + \mathbf{a}_n \tag{5}$$

The roll command ϕ_c is given by

$$\tan\phi_c = \frac{a_m \cos\phi - a_n \sin\phi}{a_h - a_m \sin\phi - a_n \cos\phi} \tag{6}$$

The magnitude of the acceleration command is

$$a_c = [(a_m \cos\phi - a_n \sin\phi)^2 + (a_h - a_m \sin\phi - a_n \cos\phi)^2]^{1/2} \tag{7}$$

This is the input to the pitch autopilot. If it is applied before $\phi = \phi_c$, it will be in error. However, since the missile response in roll is generally much faster than that in pitch, the error is relatively small. The general arrangement is shown in Fig. 4. The yaw autopilot is controlled to zero acceleration.

Simulation Results

A digital simulation was made to investigate the foregoing concepts. The simulation included a tail-controlled missile with six degrees-of-freedom, pitch, roll, and yaw autopilots and aerodynamics such that it had negligible yaw maneuver capability. Realistic limits were placed on tail deflections and rates. A point-mass target was simulated.

Some results for a constant-speed surface target with missile/target speed ratio of approximately 10 to 1 are shown in Fig. 5. The missile was flying at a constant altitude h_{nom} when the guidance computer was switched on. Since, in this particular application, it was desired to have the missile fly at constant altitude until the final run-in on the target, the guidance command was biased initially so as to make the target appear to be, approximately, at the same elevation as the missile. While it was not absolutely necessary to do this, it prevented needless competition between the guidance and the altitude control loops. As a result, $\mathbf{a}_g$ lay in the horizontal plane, that is, perpendicular to μ in Fig. 3 while the value of a_h depended upon the error signal in the altitude control loop as shown in Fig. 4. The initial heading error was such that the roll command limit ϕ_c (max) was reached. The missile rolled and the resulting normal acceleration rapidly brought the error close to zero. The induced side slip angle (not shown) was within acceptable limits. At a preprogramed distance from the target, the true elevation of the latter was indicated in the guidance command. At the same point, the altitude control feedback was switched off ($K = 0$ in Fig. 4) so that a_h became $(k \cdot \mu)$ g's. The missile then homed in on the target with a miss distance from its center of mass that would have corresponded to a hit on any target of practical dimensions. The final transients in the traces were due to the rapid change in line of sight direction because of the crossing target and the maneuver limitations of the missile.

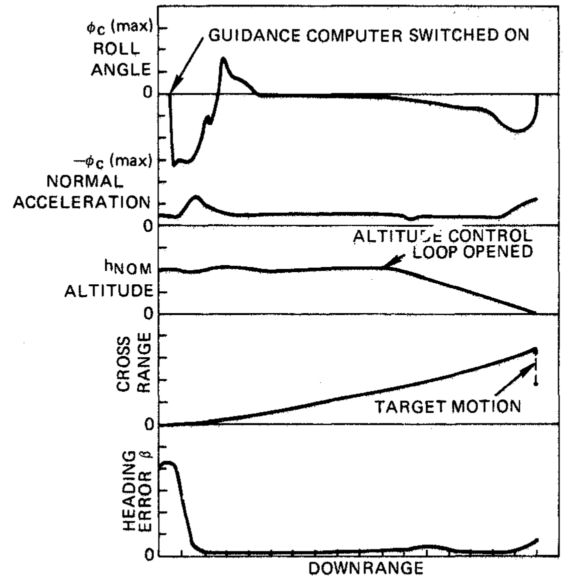

Fig. 5 Typical simulation results.

In the abovementioned example, capture was facilitated by the favorable missile/target speed ratio and relative maneuver capabilities. In general, since both the missile and target have minimum radii of curvature in their maneuvers, there will be cases where capture does not occur. The end game may be studied on the simulation by varying the critical parameters: missile/target speed ratio, maneuver curvature ratio, initial conditions, and target dimensions. An alternative analytical

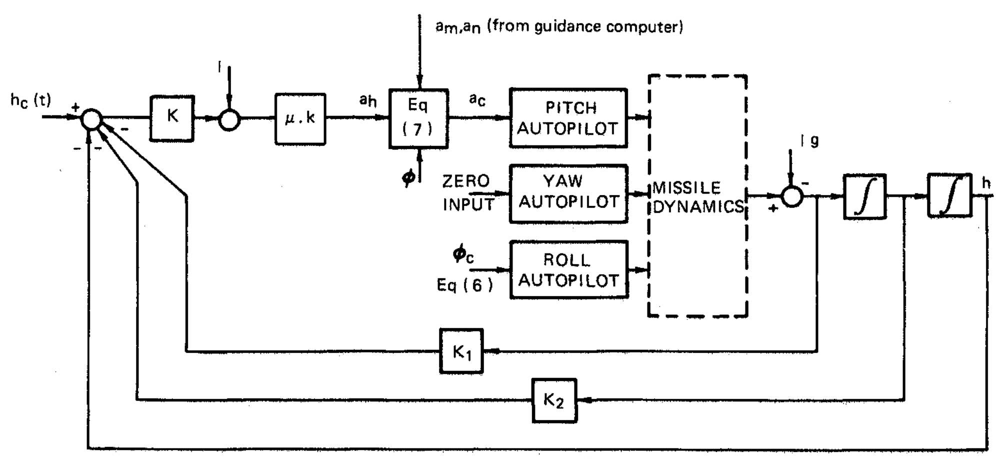

Fig. 4 Missile altitude control and bank-to-turn commands.

approach would consider the end game as a differential game similar to Isaacs' "game of two cars"[3] with payoff being miss distance. The missile would maneuver so as to minimize the payoff while the target would try to maximize it. If either played nonoptimally, the other would achieve a better payoff.

The end game study is, however, outside the scope of this Note.

References

[1] Adler, F. P., "Missile Guidance by Three-Dimensional Proportional Navigation," *Journal of Applied Physics*, Vol. 27, No. 5, May 1956, pp. 500–507.

[2] Miele, A., *Flight Mechanics*, Vol. 1, Addison-Wesley, Reading, Mass., 1962, Chap. 4.

[3] Isaacs, R., *Differential Games*, Wiley, New York, 1965.

Assessment of an Active Thermal Control Louvre Array for Spacecraft

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Louvre Array Thermal Design

LOUVRE arrays are used on spacecraft mainly for the control of instrument cabin temperature and work on the principle of opening and closing of louvre blades to suit environmental and spacecraft heat load conditions.[1,2]

The test model shown in Fig. 1 consists of three louvres supported at each end by stainless steel pins in P.T.F.E. bearings and mounted such that this combination is above a heated honeycomb panel which serves as the main radiator surface and simulates conditions of heat generation in the spacecraft. The louvres are actuated by flat spiral bimetallic springs mounted at one end of the louvre blades, shown in Fig. 1. The springs or actuators uncoil with rise in temperature of the actuator housing causing the louvre blades to open and ventilate excess heat to black space by radiative heat transfer. A fall in temperature of the actuator housing leads to the eventual closure of the louvre blades thereby isolating the spacecraft interior from black space and conserving the heat generated inside the spacecraft. The louvre array together with the spiral spring actuator and housing form an elaborate temperature control of spacecraft temperature. As heat transfer from the spacecraft to black space is by radiation the emissivity of the louvre array is an important factor in its thermal design. In the fully closed position the emissivity of the array is low being that of the polished aluminium blades. In the fully open position, the emissivity of the array is high being that of the matt black radiator base, which simulates conditions of heat generation inside the spacecraft. For intermediate blade positions the emissivity of the louvre array is calculated in the following manner. For unit depth of louvre array: total base area $= 6L$; area of base exposed normal to surface $= 6L(1-\cos\theta)$; area of louvre exposed normal to surface $= 6L\cos\theta$. Thus the effective emissivity is

$$\varepsilon = \frac{6L(1-\cos\theta)}{6L}\times 0.8+\frac{6L\cos\theta}{6L}\times 0.5 \tag{1}$$

where the significance of L and θ is clearly illustrated in Fig. 1.

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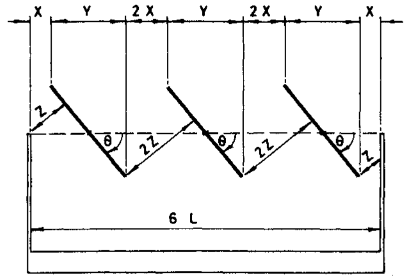

Fig. 1 The geometry of the louvre array.

However, a more accurate assessment of the exposed base area would be obtained by taking the normal distance between the louvres, i.e., the Z dimension in Fig. 1 giving the area of base exposed as $6L\sin\theta$ and

$$\varepsilon = \frac{6L\sin\theta}{6L}\times 0.8+\frac{6L\cos\theta}{6L}\times 0.5$$

i.e.

$$\varepsilon = 0.8\sin\theta+0.5\cos\theta \tag{2}$$

assuming emissivity of the matt black surface as 0.8 and emissivity of the louvres as 0.5. Equations (1) and (2) give the relationship between the effective emissivity of the system and the angular position of the blades. For temperature control purposes, the louvres are made to respond to the radiator base temperature which in effect is the temperature inside the spacecraft. At a base temperature of 10°C the actuators were positioned such that the louvre blades were in the fully closed ($\theta = 0°$) position. The actuators responded to any increase in base temperature, opening the louvres, to a maximum ($\theta = 90°$) at a base temperature of 25°C. The temperature control system was designed to be fully responsive to a temperature change of only 15°C.

Spiral Spring Actuators

The bimetallic strip from which the spiral spring actuators are made give sufficient expansion to cause rotation of the louvre blades through 90° for a temperature change of 15°C.

The spring material chosen was a nickel/iron alloy readily available. Two factors important in spiral spring design are θ, the angular rotation of the spiral spring spindle, and R the restraining torque and are given by Martin and Yarworth[3] as follows:

$$\theta = \frac{130Kl(\Delta T)_M}{t} \tag{3}$$

and

$$R = 0.189KEBt^2(\Delta T)_F \tag{4}$$

R is of the order of 0.005 oz in. as determined from bearing data and louvre dimensions.

The design values for l, t, and B are determined from Eqs. (3) and (4) bearing in mind the availability of the bimetallic strip from which the spiral springs are made. The values of these dimensions are $t = 0.012$ in., $B = 0.118$ in., and $l = 42.5$ in. Weaker springs were considered, but these were too easily damaged by handling.

Test Model Modification

A parameter of some importance in assessing the performance of louvre arrays is the open/closed power rejection ratio, a low value of the parameter implying in general an inefficient louvre-array system. An initial louvre array gave an open/closed power rejection ratio of 3:1 which was lower than desired for space application. Heat losses were excessively high and there was little thermal coupling between the base and the actuator. The thermal coupling by conduction through the spiral spring was clearly ineffective. The modifications carried out on a later model of the louvre array were as follows.